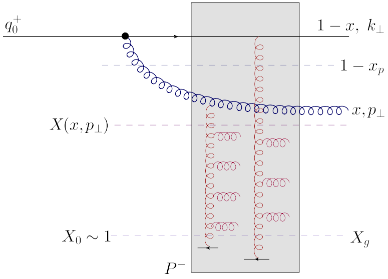
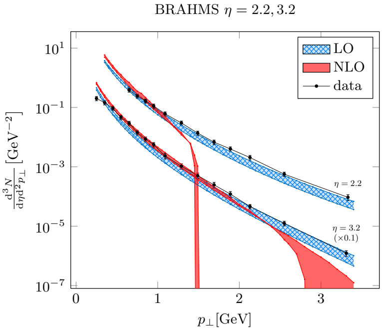
Particle production in pA collisions beyond leading order

Edmond Iancu
IPhT Saclay & CNRS
w/ A.H. Mueller and D.N. Triantafyllopoulos, arXiv:1608.05293





Introduction

- pQCD at high-energy, or 'small-x', is complicated by non-linear effects associated with the high gluon densities
  - gluon saturation, multiple scattering
  - the "power-suppressed corrections" are now effects of $\mathcal{O}(1)$
  - pQCD resummations based on eikonal approximation
  - Wilson lines, Color Glass Condensate
  - new scattering operators: dipole, quadrupole, ...
  - related to the small-x limit of TMD's
  - non-linear evolution equations: BK, B-JIMWLK
  - new factorization scheme(s): CGC (or 'generalized k_T'), hybrid

Introduction

- Realistic phenomenology requires (at least) NLO accuracy
- The CGC formalism has recently been promoted to NLO ☺
  - inclusion of running coupling corrections in BK
    (Kovchegov and Weigert, 2016; Balitsky, 2016)
  - NLO versions for the BK and B-JIMWLK equations
    (Balitsky and Chirilli, 2008, 2013; Kovner, Lublinsky, and Mulian, 2013)
  - NLO impact factor for particle production in pA collisions
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  - NLO impact factor for DIS
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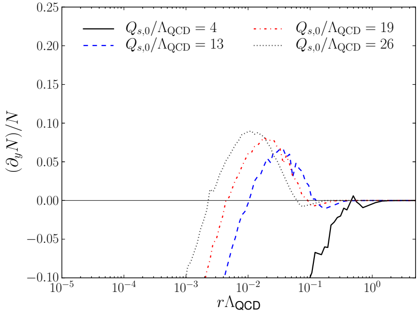
- But the NLO approximations turned out to be disappointing ☹
- New resummations (of the perturbative expansion) have been recently devised to cure these problems

NLO BK evolution

- "Negative growth" of the dipole scattering amplitude



Lappi, Mäntysaari, arXiv:1502.02400

- Hardly a surprise
  - similar problems for NLO BFKL
  - large transverse logarithms
  - collinear resummations
  - Mellin representation

(Salam, Ciafaloni, Colferai, Stasto, 98-03; Altarelli, Ball, Forte, 00-03)

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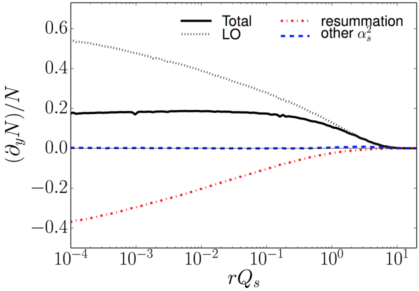

Lappi, Mäntysaari, arXiv:1601.06598

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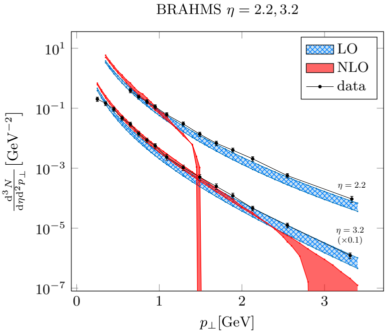
- Collinear improvement for NLO BK (transverse coordinates)

  (E.I., J. Madrigal, A. Mueller, G. Soyez, and D. Triantafyllopoulos, 2015)

- Evolution becomes stable with promising phenomenology
  - excellents fits to DIS *(Iancu et al, 2015; Albacete, 2015)*

Particle production in d+Au collisions (RHIC)

- Very good agreement at low $p_\perp$ ☺ ... but negative at larger $p_\perp$ ☹



Stasto, Xiao, and Zaslavsky, arXiv:1307.4057

- Is this a real problem ?
  - "small-x resummations do not apply at large $p_\perp$"
  - but $p_\perp \sim Q_s$ is not that large !
  - and the turn-over is dramatic
- Are the 2 problems related ?
  - transverse logs are ubiquitous

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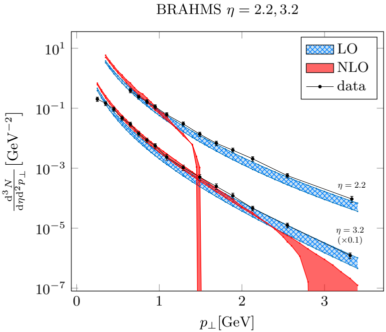

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- Various proposals which alleviate the problem (pushed to higher $p_\perp$)
  - *Kang, Vitev, and Xing, arXiv:1403.5221*
  - *Altinoluk, Armesto, Beuf, Kovner, and Lublinsky, arXiv:1411.2869*
  - *Ducloué, Lappi, and Zhu, arXiv:1604.00225*

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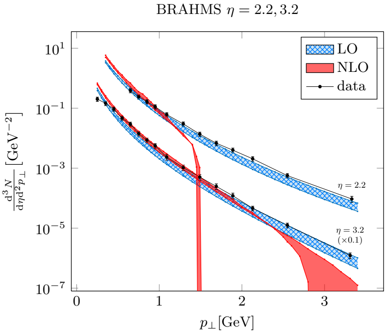

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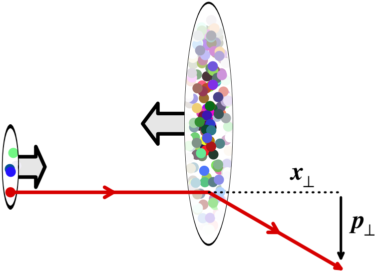
- A fresh look at the NLO calculation of the cross-section

  (E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)

Quark production at forward rapidity

- A quark initially collinear with the proton acquires a transverse momentum $p_\perp$ via multiple scattering off the dense nucleus



$$\eta = \frac{1}{2}\ln\frac{p^+}{p^-}$$

$$x_p = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{\eta}$$

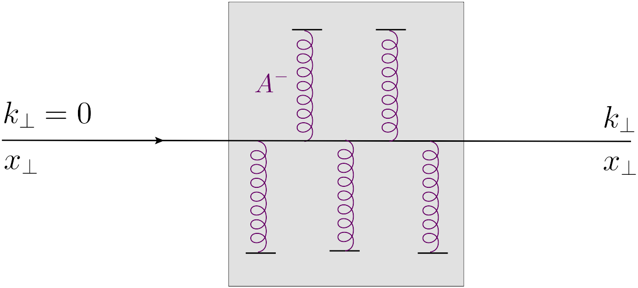
$$X_g = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{-\eta}$$

  - η : quark rapidity in the COM frame
  - x_p : longitudinal fraction of the quark in the proton
  - X_g : longitudinal fraction of the gluon in the target

- $\eta > 1$: 'forward rapidity' $\Longrightarrow X_g \ll x_p$ ('dense-dilute')
  - RHIC: $p_\perp = 2$ GeV, $\eta = 3 \Longrightarrow x_p = 0.2$ & $X_g = 5 \times 10^{-4}$

Wilson lines

- Multiple scattering can be resummed in the eikonal approximation



Amplitude: $$\mathcal{M}_{ij}(\boldsymbol{k}_\perp) \equiv \int \mathrm{d}^2\boldsymbol{x}_\perp\, \mathrm{e}^{-i\boldsymbol{x}_\perp\cdot\boldsymbol{k}_\perp}\, V_{ij}(\boldsymbol{x}_\perp)$$

Wilson line: $$V(\boldsymbol{x}_\perp) = \mathrm{P}\exp\left\{ig\int \mathrm{d}x^+ A_a^-(x^+,\boldsymbol{x}_\perp)t^a\right\}$$

- A_a^- : color field representing small-x gluons in the nucleus

Wilson lines

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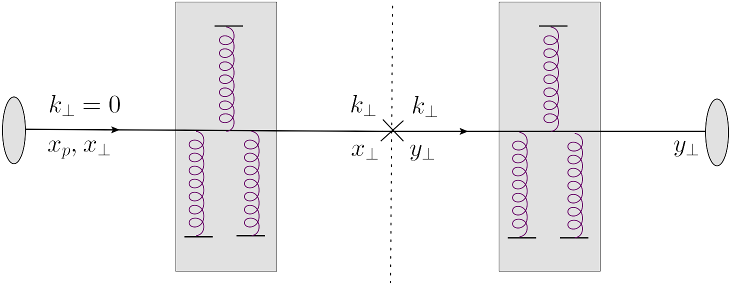

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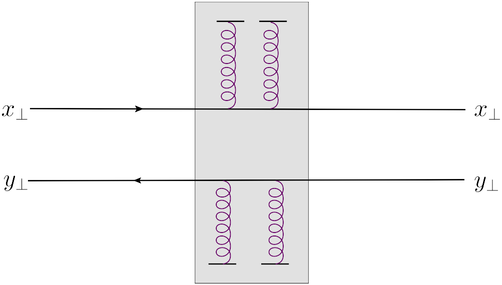
Cross-section: $$\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}^2\boldsymbol{k}_\perp} \simeq x_p q(x_p, Q^2)\, \frac{1}{N_c} \left\langle \sum_{ij} |\mathcal{M}_{ij}(\boldsymbol{k}_\perp)|^2 \right\rangle_{X_g}$$

- Average over the color fields A^- in the target (CGC)

Dipole picture

- Equivalently: the elastic S-matrix for a $q\bar{q}$ color dipole



$$S(\boldsymbol{x}, \boldsymbol{y}; X_g) \equiv \frac{1}{N_c} \left\langle \mathrm{tr}\left[V(\boldsymbol{x})V^\dagger(\boldsymbol{y})\right]\right\rangle_{X_g}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}^2\boldsymbol{k}} \simeq x_p q(x_p) \int_{\boldsymbol{x},\boldsymbol{y}} \mathrm{e}^{-\mathrm{i}(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{k}}\ S(\boldsymbol{x}, \boldsymbol{y}; X_g)$$

- The Fourier transform $\mathcal{S}(\boldsymbol{k}, X_g)$ of the dipole S-matrix is a TMD for gluons in the nucleus *(cf. talk by Daniel Boer)*.

Dipole picture

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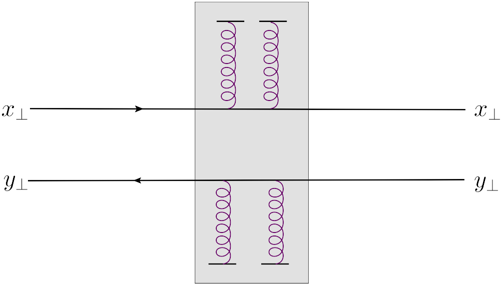

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- 'Hybrid factorization': collinear-fact. for p & CGC-fact. for A *(Dumitru, Hayashigaki, and Jalilian-Marian, arXiv:hep-ph/0506308)*.

Dipole picture

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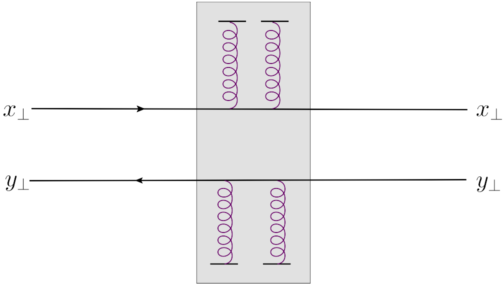

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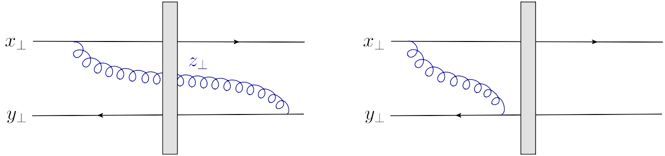
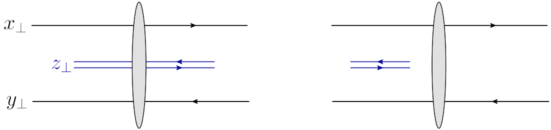
- The dipole picture is preserved by the high-energy evolution up to NLO *(Kovchegov and Tuchin, 2002; Mueller and Munier, 2012)*

- Probability $\sim \alpha_s \ln \frac{1}{x}$ to radiate a soft gluon with $x \equiv \frac{p^+}{q_0^+} \ll 1$



- Large N_c : the dipole splits into two new dipoles *(Al Mueller, 1990)*



- Evolution equation for the dipole S–matrix $S_{\boldsymbol{xy}}(Y)$ with $Y \equiv \ln(1/x)$

$$\frac{\partial S_{\boldsymbol{xy}}}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int \mathrm{d}^2\boldsymbol{z} \, \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} \left[S_{\boldsymbol{xz}}S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}}\right]$$

The BK equation *(Balitsky, '96; Kovchegov, '99)*

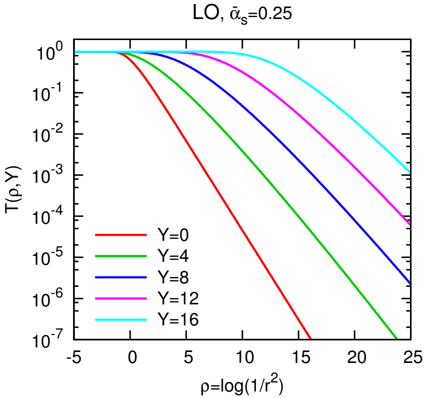
- Non-linear equation for the scattering amplitude $T_{\boldsymbol{xy}} \equiv 1 - S_{\boldsymbol{xy}}$

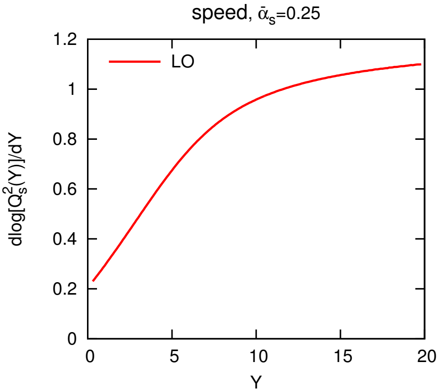
$$\frac{\partial T_{\boldsymbol{xy}}}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int \mathrm{d}^2\boldsymbol{z}\, \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} \left[T_{\boldsymbol{xz}} + T_{\boldsymbol{zy}} - T_{\boldsymbol{xy}} - T_{\boldsymbol{xz}}T_{\boldsymbol{zy}}\right]$$

- Non-linear generalization of the BFKL equation
- 2 regimes depending upon the parent dipole size $r = |\boldsymbol{x} - \boldsymbol{y}|$
  - small dipole $r \ll 1/Q_s(Y)$: weak scattering $T \ll 1 \Rightarrow$ BFKL equation
  - larger dipole $r \gtrsim 1/Q_s(Y)$: approach to black disk limit $T = 1$
  - saturation momentum $Q_s(Y)$: $T(r, Y) = 0.5$ when $r = 1/Q_s(Y)$
- $Q_s(Y)$ increases rapidly with Y due to the BFKL dynamics
  - successive soft emissions leading to an exponential growth of T

The saturation front

- $T \equiv 1 - S$ as a function of $\rho \equiv \ln(1/r^2)$ with increasing Y

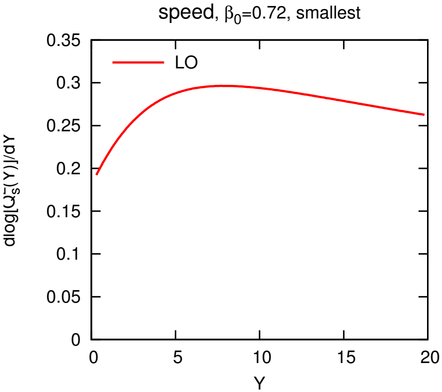


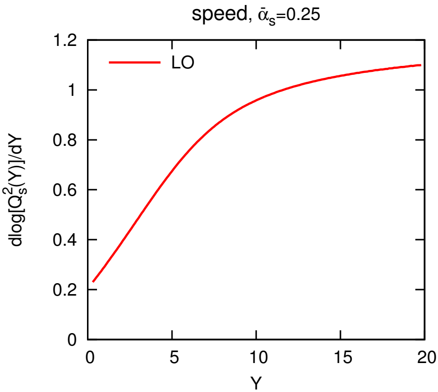


- color transparency at large ρ (small r) : $T \propto r^2 = \mathrm{e}^{-\rho}$
- unitarization at small ρ (large r) : $T = 1$ (black disk limit)
- saturation exponent: $\lambda_s \equiv \frac{\mathrm{d}\ln Q_s^2}{\mathrm{d}Y} \simeq 1$ for $Y \gtrsim 10$

Adding running coupling: rcBK

- The LO saturation exponent is way too large: $\lambda_{\rm HERA} = 0.2 \div 0.3$



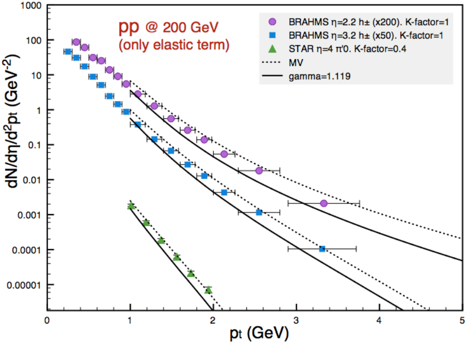


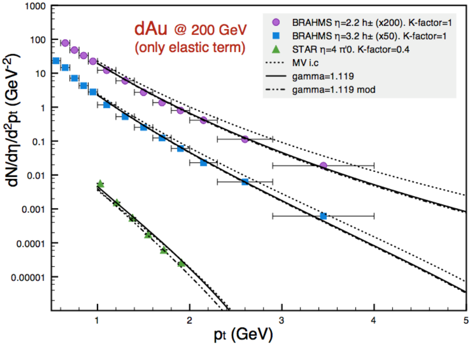
- Including running coupling dramatically slows down the evolution
  - realistic value for the saturation exponent
- ... but there are other, equally important, NLO corrections !

LO phenomenology (rcBK)

(Albacete, Dumitru, Fujii, Nara, arXiv:1209:2001)

- Fit parameters: initial condition for the rcBK equation + K-factors

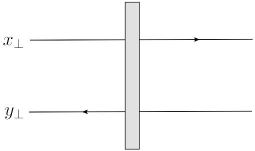


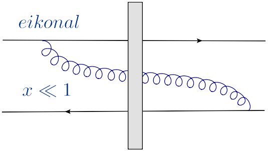


$$\frac{\mathrm{d}N}{\mathrm{d}y\,\mathrm{d}^2\boldsymbol{k}}\Big|_{\mathrm{LO}} = K^h \int_{x_p}^1 \frac{\mathrm{d}z}{z^2}\,\frac{x_p}{z} q\left(\frac{x_p}{z}\right)\,\mathcal{S}\left(\frac{\boldsymbol{k}}{z}, X_g\right)\,D_{h/q}(z)$$

Beyond leading order

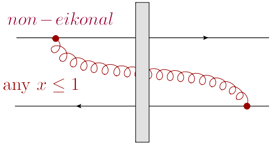
- LO approximation: any number $n \geq 0$ of soft emissions $\Longrightarrow$ $(\alpha_s Y)^n$





$= \mathcal{O}(\alpha_s Y)$

- NLO correction to impact factor: the first gluon is hard



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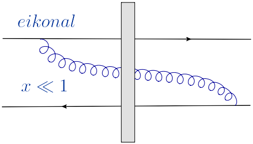
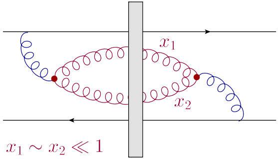
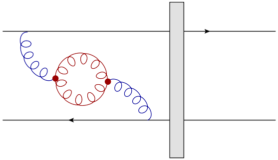


$= \mathcal{O}(\alpha_s)$

- NLO corrections to the evolution: 2 soft gluons, with similar values of x





$= \mathcal{O}(\alpha_s^2 Y)$

NLO factorization scheme by CXY

(Chirilli, Xiao, and Yuan, arXiv:1203.6139 [hep-ph])

- Recall first the factorization at LO :

$$\frac{\mathrm{d}N}{\mathrm{d}y\,\mathrm{d}^2\boldsymbol{k}}\Big|_{\rm LO} = x_p q(x_p) \int \mathrm{d}^2\boldsymbol{r}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{r}\cdot\boldsymbol{k}}\, S_{\rm LO}(\boldsymbol{r}, X_g) \equiv \mathcal{S}_{\rm LO}(\boldsymbol{k}, X_g)$$

- CXY: "Replace $\mathcal{S}_{\rm LO}$ by $\mathcal{S} \equiv \mathcal{S}_{\rm NLO}$ and add the impact factor correction"

$$\frac{\mathrm{d}N}{\mathrm{d}y\,\mathrm{d}^2\boldsymbol{k}}\Big|_{\rm NLO} = \mathcal{S}(\boldsymbol{k}, X_g) + \bar{\alpha}_s \int_0^1 \frac{\mathrm{d}x}{x}\left[\mathcal{K}(x) - \mathcal{K}(0)\right] \mathcal{S}(\boldsymbol{k}, X_g)$$

  - $\mathcal{K}(x)$: kernel for emitting a gluon by the dipole with exact kinematics
  - $\mathcal{K}(0)$: small-x (eikonal) limit of $\mathcal{K}(x)$ = dipole kernel
  - 'plus' prescription for the integral over x
  - local in x (here X_g)

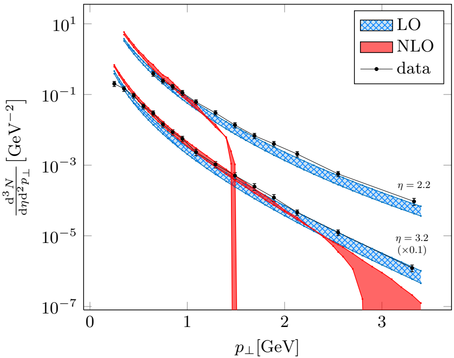
- Natural generalization of NLO $k_\perp$-factorization to high density

The negativity problem

(Stasto, Xiao, and Zaslavsky, arXiv:1307.4057)

- Sudden drop in the numerical estimate at momenta $p_\perp$ of order Q_s

BRAHMS $\eta = 2.2, 3.2$



- "NLO evolution is notoriously unstable"
- Sure, but in this calculation $\mathcal{S} \approx \mathcal{S}_{\rm rcBK}$
  - rcBK evolution is well behaved
  - the actual "LO approx" in practice

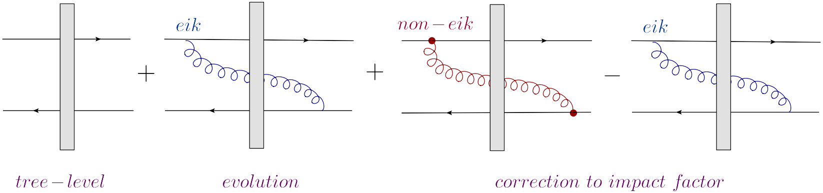
$$\left.\frac{dN}{dy\, d^2\boldsymbol{k}}\right|_{\rm LO} = \mathcal{S}_{\rm rcBK}(\boldsymbol{k}, X_g)$$

- The NLO correction to the impact factor is negative (not a real surprise) ... and dominates over the LO result at sufficiently large $k_\perp$

Deconstructing the NLO approximation

(E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)

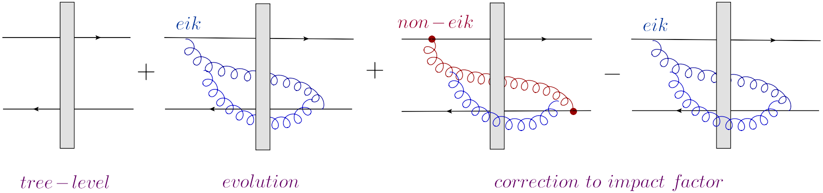
- One gluon emission



Deconstructing the NLO approximation

(E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)

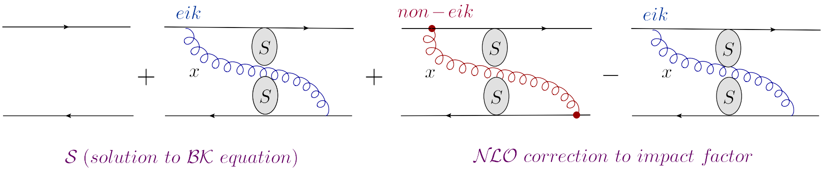
- Building up the evolution ...



Deconstructing the NLO approximation

(E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)

- LO evolution (say, rcBK) fully included



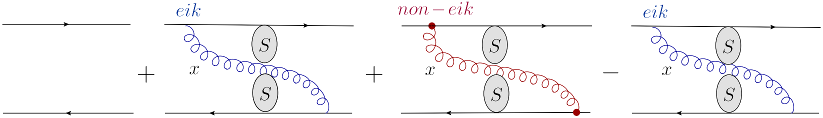
$$\mathcal{N} = \mathcal{S}(\boldsymbol{k}, X_g) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \left[\mathcal{K}(x) - \mathcal{K}(0)\right] \mathcal{S}\big(\boldsymbol{k}, X(x)\big)\,; \quad X(x) \equiv \frac{X_g}{x}$$

- Non-local in x: target evolution depends upon the x-value of the gluon
  - lower limit on x: $X \leq 1 \Longrightarrow x \geq X_g$
- Different from the CXY formula ... but equivalent to NLO accuracy
  - $\mathcal{S}\big(X(x)\big) \simeq \mathcal{S}(X_g)$ since integral controlled by $x \sim 1$
  - remove lower limit $X_g \Longrightarrow$ the 'plus' prescription

Deconstructing the NLO approximation

(E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)

- LO evolution (say, rcBK) fully included

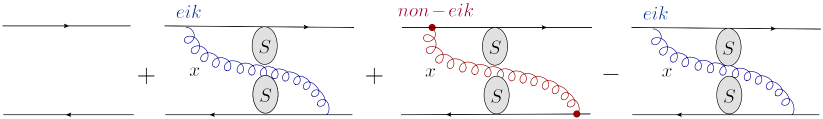


$\mathcal{S}$ (solution to $\mathcal{BK}$ equation) — $\mathcal{NLO}$ correction to impact factor

$$\mathcal{N} = \mathcal{S}(\boldsymbol{k}, X_g) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \left[\mathcal{K}(x) - \mathcal{K}(0)\right] \mathcal{S}\big(\boldsymbol{k}, X(x)\big)\,; \quad X(x) \equiv \frac{X_g}{x}$$

- Non-local in x: target evolution depends upon the x-value of the gluon
  - lower limit on x: $X \leq 1 \Longrightarrow x \geq X_g$
- Different from the CXY formula ... but equivalent to NLO accuracy
- N.B.: $\mathcal{S}(X_g) > \mathcal{S}\big(X(x)\big)$ for any $x < 1 \Longrightarrow$ some over-subtraction

- One adds and subtracts the LO evolution (the dominant contribution !)



$\mathcal{S}$ (solution to $\mathcal{BK}$ equation)

$\mathcal{NLO}$ correction to impact factor

$$\mathcal{N} \;=\; \mathcal{S}(\boldsymbol{k}, X_g) + \bar{\alpha}_s(k_\perp^2)\int_{X_g}^{1}\frac{\mathrm{d}x}{x}\,\big[\mathcal{K}(x) - \mathcal{K}(0)\big]\,\mathcal{S}\big(\boldsymbol{k}, X(x)\big)$$

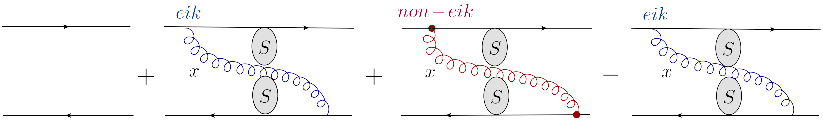
- The 'added' and 'subtracted' pieces are treated differently
  - the 'added' piece is used to reconstruct the solution to BK

$$\mathcal{S}(\boldsymbol{k}, X_g) \;=\; \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s\int_{X_g}^{1}\frac{\mathrm{d}x}{x}\,\mathcal{K}(0)\,\mathcal{S}\big(\boldsymbol{k}, X(x)\big)$$

  - the 'subtracted' piece is used to isolate the NLO impact factor

The fine-tuning problem

- One adds and subtracts the LO evolution (the dominant contribution !)



$$\mathcal{N}_{\rm CXY} \;=\; \mathcal{S}(\boldsymbol{k}, X_g) + \bar{\alpha}_s(k_\perp^2)\int_0^1 \frac{\mathrm{d}x}{x}\,\big[\mathcal{K}(x) - \mathcal{K}(0)\big]\,\mathcal{S}\big(\boldsymbol{k}, X_g\big)$$

- The ‘added’ and ‘subtracted’ pieces are treated differently
  - the ‘added’ piece is used to reconstruct the solution to BK

$$\mathcal{S}(\boldsymbol{k}, X_g) \;=\; \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s\int_{X_g}^1 \frac{\mathrm{d}x}{x}\,\mathcal{K}(0)\,\mathcal{S}\big(\boldsymbol{k}, X(x)\big)$$

  - the ‘subtracted’ piece is used to isolate the NLO impact factor
- CXY: the subtraction is performed only approximately

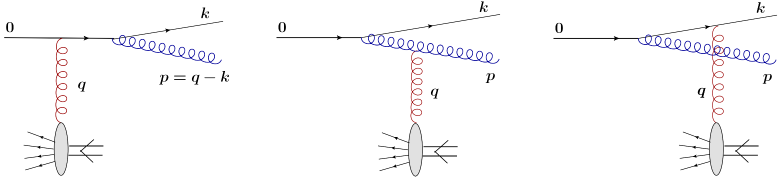
Why is this a problem ?

- Any approximation/numerical error in the BK solution or in the subtraction procedure $\Longrightarrow$ mismatch between the 'added' and 'subtracted' pieces

- An extreme example: the GBW saturation model

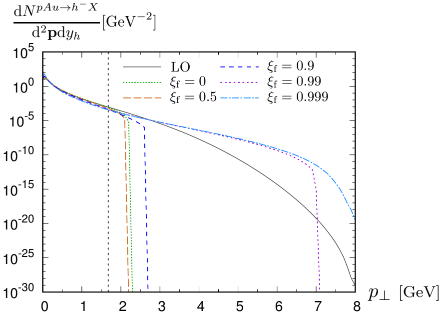
$$\mathcal{S}_{\rm GBW}(\boldsymbol{k}, X) \,\propto\, {\rm e}^{-\frac{k_\perp^2}{Q_s^2}}$$

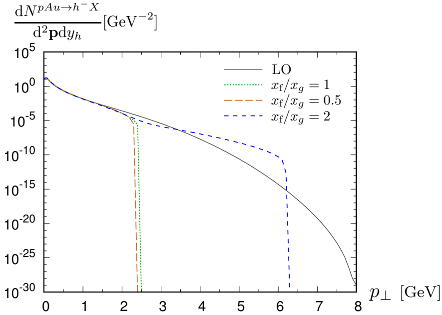
  - the 'added' piece is exponentially suppressed at $k_\perp \gg Q_s$
  - the 'subtracted' piece develops a power-law tail $\propto 1/k_\perp^4$



  - the overall result becomes negative at sufficiently large $k_\perp$

CXY factorization + GBW model for $\mathcal{S}$





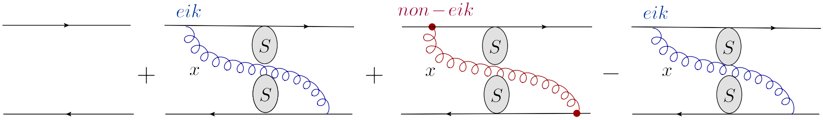
(Ducloué, Lappi, and Zhu, arXiv:1604.00225)

- This behavior is indeed visible in the numerical results
- Rapidity factorization scale $x_0 \equiv 1 - \xi_f$
- Decreasing x_0 pushes the problem to higher $k_\perp$
  - strongly dependent upon the precise implementation of x_0

Back to basics

(E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)

- Why do we need the rapidity subtraction in the first place ?
  - to disentangle evolution from corrections to the impact factor
  - to ensure a strict expansion in powers of α_s



$\mathcal{S}$ (solution to $\mathcal{BK}$ equation) $\mathcal{NLO}$ correction to impact factor

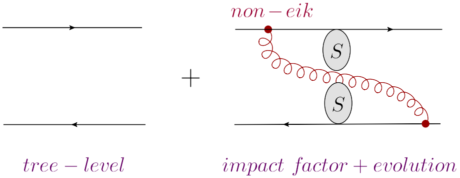
$$\mathcal{N} = \mathcal{S}(\boldsymbol{k}, X_g) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \left[\mathcal{K}(x) - \mathcal{K}(0)\right] \mathcal{S}\big(\boldsymbol{k}, X(x)\big)$$

- All that is fine ... so long as it works !

A new factorization scheme

(E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)

- But when it doesn't, better return to the skeleton structure of pQCD



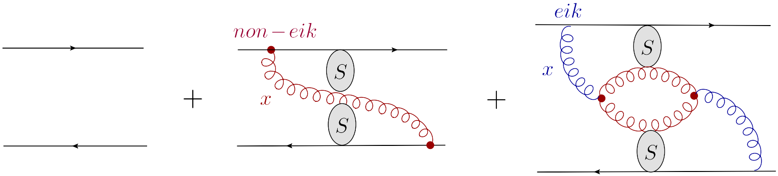
$$\mathcal{N}(\boldsymbol{k}) = \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s(k_\perp^2) \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \mathcal{K}(x) \, \mathcal{S}\big(\boldsymbol{k}, X(x)\big)$$

- NLO corrections to impact factor and evolution are mixed with each other
  - it goes beyond a strict NLO approximation
  - non-local in x : goes beyond $k_\perp$-factorization
- Positive definite by construction
  - with $\mathcal{K}(x) \to \mathcal{K}(0)$: the r.h.s. of the LO BK equation

Restoring full NLO evolution

(E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)

- Recall: the NLO BK evolution also involves 2-loop graphs



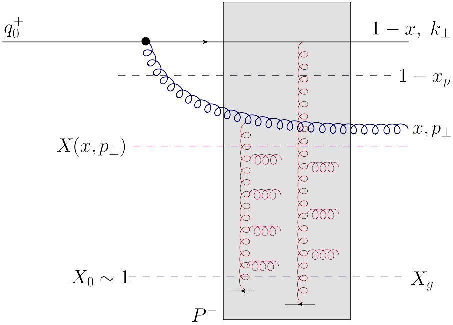
tree − level   *impact factor + $\mathcal{LO}$ evolution*   *$\mathcal{NLO}$ evolution*

$$\mathcal{N} = \mathcal{S}_0 + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x}\, \mathcal{K}(x)\, \mathcal{S}\big(X(x)\big) + \bar{\alpha}_s^2 \int_{X_g}^{1} \frac{\mathrm{d}x}{x}\, \mathcal{K}_2(0)\, \mathcal{S}\big(X(x)\big)$$

- $\mathcal{K}_2(0)$: NLO correction to the BK kernel with collinear improvement
- Complicated in practice ... but one can start with $\mathcal{S} \approx \mathcal{S}_{\rm rcBK}$ and $\mathcal{K}_2 = 0$
  - remember: the problem already shows up with the LO evolution

Exact kinematics for target evolution

- 'Real amplitude' : the gluon is produced in the final state



- LC energy conservation:

$$\frac{k_\perp^2}{2(1-x)q_0^+} + \frac{p_\perp^2}{2xq_0^+} = XP^-$$

- $\Longrightarrow \quad X = X(x, p_\perp)$
- simplifies when $k_\perp \simeq p_\perp \gg Q_s$

$$X(x) \simeq \frac{k_\perp^2}{xs} = \frac{X_g}{x}$$

- $X \leq 1 \Longrightarrow x \geq X_g$

- Equivalently: gluon lifetime should be larger than the target width
- The same condition holds for the 'virtual' corrections
  - non-trivial cancellations required by probability conservation

Some proposals to solve the problem

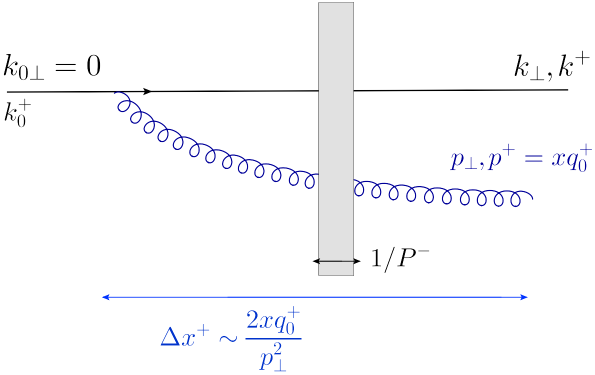
- General idea: the 'subtracted' term performs an ... over-subtraction
- Strategy: reduce the longitudinal (x) phase-space for the 'hard' gluon
  - factorization scale x_0 separating 'evolution' from 'impact factor'
    (Kang, Vitev, and Xing, arXiv:1403.5221)

$$\int_0^{1} \frac{\mathrm{d}x}{x}\left[\mathcal{K}(x) - \mathcal{K}(0)\right] \Longrightarrow \int_0^{x_0} \frac{\mathrm{d}x}{x}\left[\mathcal{K}(x) - \mathcal{K}(0)\right]$$

  - x_0 can depend upon $k_\perp$, say to account for 'time-ordering'
    (Ducloué, Lappi, and Zhu, arXiv:1604.00225)
- In principle, it shouldn't matter that much
  - the x_0–dependence must cancel in a complete calculation
- In practice, it only pushes the problem up to somewhat higher $k_\perp$
  - also, strongly dependent upon the precise implementation of x_0

(Altinoluk, Armesto, Beuf, Kovner, and Lublinsky, arXiv:1411.2869)

- x cannot be arbitrarily small since constrained by energy conservation



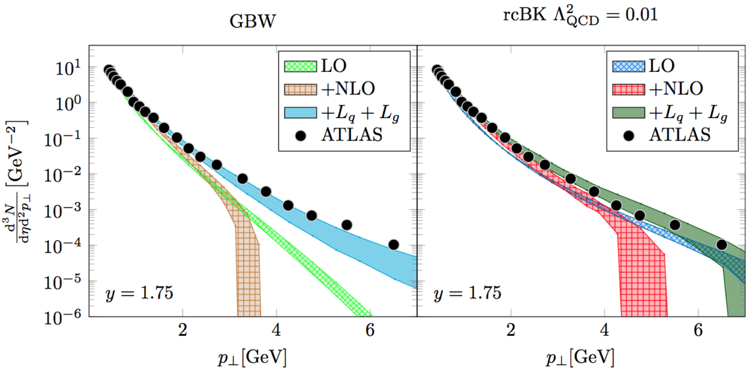
- Gluon lifetime should be larger than the target width

$$\frac{2xq_0^+}{p_\perp^2} > \frac{1}{P^-} \Longrightarrow x > \frac{p_\perp^2}{s}$$

Implementing the constraint

(Watanabe, Xiao, Yuan, and Zaslavsky, arXiv:1505:05183)

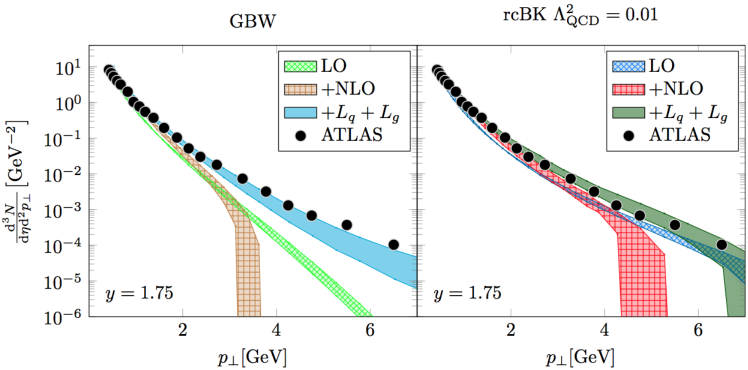
- It matters for the subtraction scheme only if $k_\perp \gg p_\perp$



- ... which means it doesn't really matter in practice
  - when $k_\perp \gtrsim Q_s$, one also has $k_\perp \sim p_\perp$

(Watanabe, Xiao, Yuan, and Zaslavsky, arXiv:1505:05183)

- It matters for the subtraction scheme only if $k_\perp \gg p_\perp$



- Once again, it pushes the problem to higher $k_\perp$
  - ... and strongly dependent upon the model/evolution chosen for $\mathcal{S}$